

# 6671 Pure Mathematics P1

## Mark Scheme

Question Number	Scheme	Marks
1.	<p>Forming equation in <math>x</math> or <math>y</math> by attempt to eliminate one variable</p> $(3 - y)^2 + y = 15 \text{ or } x^2 + (3 - x) = 15$ $y^2 - 5y - 6 = 0 \text{ or } x^2 - x - 12 = 0 \text{ (Correct 3 term version)}$ <p><u>Attempt at solution</u> i.e. solving 3 term quadratic: <math>(y - 6)(y + 1) = 0</math>, <math>y = \dots</math>  or <math>(x - 4)(x + 3) = 0</math>, <math>x = \dots</math></p> <p>or correct use of formula or correct use of completing the square</p> $x = 4 \text{ and } x = -3 \text{ or } y = -1 \text{ and } y = 6$ <p>Finding the values of the other coordinates (M attempt one, A both)</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>M1 A1 ft</p> <p style="text-align: right;">(6)</p>
2.	<p>Using <math>\sin^2 \theta + \cos^2 \theta = 1</math> to give a quadratic in <math>\cos \theta</math>.</p> <p>Attempt to solve <math>\cos^2 \theta + \cos \theta = 0</math></p> $(\cos \theta = 0) \Rightarrow \theta = \frac{\pi}{2}, \frac{3\pi}{2}$ $(\cos \theta = -1) \Rightarrow \theta = \pi$ <p>(Candidate who writes down 3 answers only with no working scores a maximum of 3)</p>	<p>M1</p> <p>M1</p> <p>B1, B1</p> <p>B1</p> <p style="text-align: right;">(5)</p>
3.	<p>(a) Attempt <math>f(2) = 16 - 4 + 4 - 16 = 0 \Rightarrow (x - 2)</math> is a factor      must be statement for A1</p> <p>(b) <math>c = 8</math></p> <p><u>A complete method to find <math>b</math></u></p> <p>Either compare coefficients of <math>x</math> or <math>x^2</math>: <math>-2b + 8 = 2</math>, or <math>-4 + b = -1</math>  Or substitute value of <math>x</math> (may be implied): e.g. <math>(x = 1) \Rightarrow -13 = (-1)(10 + b)</math></p> $b = 3$ <p>(c) Checking <math>b^2 - 8c</math>; <math>-55 \Rightarrow</math> no real roots to the quadratic  <math>\Rightarrow x = 2</math> is the only solution</p>	<p>M1; A1</p> <p style="text-align: right;">(2)</p> <p>B1</p> <p>M1</p> <p>A1</p> <p style="text-align: right;">(3)</p> <p>M1; A1</p> <p>A1</p> <p style="text-align: right;">(3)</p>

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4 (a)	<p>Correct strategy for differentiation e.g. <math>y = 4x^2 + (5x-1)x^{-1}</math> multiplied out with correct differentiation method, or product or quotient rules applied correctly to <math>\frac{5x-1}{x}</math>.</p> $\frac{dy}{dx} = 8x, + \frac{1}{x^2}$ <p style="text-align: right;">B1 for 8x seen anywhere.</p>	<p>M1</p> <p>B1, A1 (3)</p>
(b)	<p>Putting <math>\frac{dy}{dx} = 0</math></p> <p>So <math>8x^3 + 1 = 0 \Rightarrow x = -\frac{1}{2}</math>.</p> <p style="text-align: center;">M1 requires multiplication by denominator and use of a root in the solution</p>	<p>M1</p> <p>M1 A1 (3)</p>
(c)	<p>Complete method:</p> <p>Either <math>\frac{d^2y}{dx^2} = 8 - \frac{2}{x^3}</math>, with <math>x</math> value substituted, or gradient either side checked</p> <p>Completely correct argument, either <math>&gt; 0</math> with no error seen,( 24)or <math>-ve</math> to <math>+ve</math> gradient, then <b>minimum</b> stated</p>	<p>M1</p> <p>A1 (2)</p>
5(a)	<p><math>p = 15, q = -3</math></p> <p style="text-align: right;">Special case if B0 B0, allow M1 for method, e.g. <math>8 = \frac{1+p}{2}</math></p>	<p>B1, B1 (2)</p>
(b)	<p>Gradient of line <math>ADC = -\frac{5}{7}</math>, gradient of perpendicular line = <math>-\frac{1}{\text{gradient } ADC} \left( \frac{7}{5} \right)</math></p> <p>Equation of <math>l</math>: <math>y - 2 = \left(\frac{7}{5}\right)(x - 8)</math> <math>\Rightarrow 7x - 5y - 46 = 0</math> (Allow rearrangements of this)</p>	<p>B1, M1</p> <p>M1 A1ft A1cao (5)</p>
(c)	<p>Substituting <math>y = 7</math> and finding value for <math>x</math>,</p> <p style="text-align: center;"><math>x = \frac{81}{7}</math> or <math>11\frac{4}{7}</math></p>	<p>M1</p> <p>A1 (2)</p>

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6 (a)	$P = r\theta + 2r, \quad A = \frac{1}{2}r^2\theta$	B1, B1 (2)
(b)	Substituting value for $r$ and equating $P$ to $A$ . [ $2\sqrt{2}(2+\theta) = \frac{1}{2}(2\sqrt{2})^2\theta$ ] Correct process to find $\theta$ [ $\theta(\sqrt{2}-1) = 2$ ] $\theta = \frac{2}{\sqrt{2}-1} \quad * \text{ often see } \theta = \frac{4\sqrt{2}}{4-2\sqrt{2}}$	M1 M1 A1 c.s.o. (3)
(c)	Multiply numerator and denominator by $(\sqrt{2}+1)$ $2, +2\sqrt{2}$	M1 A1, A1 (3)
7 (a)	Applying correct formula [ $325 = 120 + 5(n-1)$ ] Solving to give $n = 42$ * (or verifying in correct equation)	M1 A1 (2)
(b)	Using formula for sum of AP: $S = \frac{42}{2}\{240 + 5(42-1)\}$ or use $\frac{n}{2}\{a+l\}$ $= 9345$	M1 A1 A1 (3)
(c)	Recognising GP with $r = 0.98$ Value ( in £ ) = $7200 (0.98)^{24}$ $= 4434 \text{ ( only this value)}$	M1 M1 A1 (3)

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8 (a)	Substitute $x=0, y = \sqrt{3}$ to give $\sqrt{3} = k \frac{\sqrt{3}}{2} \Rightarrow k = 2$ (or verify result) must see $\frac{\sqrt{3}}{2}$  (b) $p = 120, \quad q = 300$ (f.t. on $p + 180$ )  (c) $\arcsin(-0.8) = -53.1$ or $\arcsin(0.8) = 53.1$  $(x + 60) = 180 - \arcsin(-0.8)$ or equivalent $180 + \arcsin 0.8$  First value of $x = 233.1 - 60$ , i.e. $x = 173.1$  OR $(x + 60) = 360 + \arcsin(-0.8)$ or equivalent $360 - \arcsin 0.8$ , i.e. $x = 246.9$	B1 (1)  B1, B1ft (2)  B1  M1  A1  M1, A1 (5)
9 (a)	$(x-3)^2, +9$ isw. $a = 3$ and $b = 9$ may just be written down with no method shown.  (b) $P$ is $(3, 9)$  (c) $A = (0, 18)$  $\frac{dy}{dx} = 2x - 6$ , at $A$ $m = -6$  Equation of tangent is $y - 18 = -6x$ (in any form)  (d) Showing that line meets $x$ axis directly below $P$ , i.e. at $x = 3$ .  (e) $A = \int x^2 - 6x + 18 dx = [\frac{1}{3}x^3 - 3x^2 + 18x]$ Substituting $x = 3$ to find area $A$ under curve $A$ [=36] Area of $R = A - \text{area of triangle} = A - \frac{1}{2} \times 18 \times 3, = 9$ Alternative: $\int x^2 - 6x + 18 - (18 - 6x) dx$ M1 $= \frac{1}{3}x^3$ M1 A1 ft Use $x = 3$ to give answer 9 M1 A1	B1, M1 A1 (3) B1 ft, B1ft (2)  B1  M1 A1 A1ft (4)  A1cso (1)  M1 A1 M1 M1 A1 (5)